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Algorithm to determine the momentum of a beam particle using beam chamber tracks and the optics of the line

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Abstract

The momentum spread (bite) of the MIPP secondary beam is controlled by the vertical variable collimator MC6CY. The beam line optics are designed to provide vertical non-zero dispersion at the point. MIPP runs with momentum spreads of the order of 2-5%. In this note we outline an algorithm to determine the momentum of an individual beam particle using the trajectory measured in the beam chambers and a knowledge of the optics of the line. The resolution obtained should be better than the momentum spread of the beamline. *The algorithm outlined is a general one to determine unknown parameters from data using theoretical knowledge of the correlations of the known and unknown parameters.*

Method:-

We generate a large number N of beam particles at a central momentum **p** and transport them to MC7 from the primary target for a particular set of beam currents consistent with p. The beam collimator is set to the nominal value. Let (x_i, x'_i), (y_i, y'_i), p_i denote the position, slope in the x-z and y-z views of the i^{th} beam particle at a value z chosen to be the average z of all the beam chamber planes. (z runs along the beam direction). To clarify, $x_i^{'} = \frac{dx}{dz}$; $y_i^{'} = \frac{dy}{dz}$ and **x,y** are the co-ordinates of the beam tracks at **z**=average **z** of all the beam planes. Then let $q_i = \frac{p_i - p}{p}$ for the i^{th} track; q_i is the fractional momentum off set of the i^{th} track from the nominal beam momentum **p** and is dimensionless.

Let us denote by a 5 component vector u_i, **i=1,5** the quantities (x_i, x'_i), (y_i, y'_i), q_i for a track. Then the error matrix E of the 5 observables is defined as

$$E_{ij} = \left\langle (u_i - \overline{u}_i)(u_j - \overline{u}_j) \right\rangle = \left\langle u_i u_j \right\rangle - \overline{u}_i \overline{u}_j$$

Where the symbols <> and bar both denote average over the ensemble N of tracks. We can evaluate this matrix using the beamline monte carlo.

Then the Hessian matrix (abbreviated as the H matrix) is defined as $H \equiv E^{-1}$ and the χ^2 for a track is defined as

$$\chi^2 = \sum_{i,j} H_{ij}(u_i - \overline{u}_i)(u_j - \overline{u}_j) = H_{ij}\lambda_i\lambda_j;$$
$$\text{where } \lambda_i \equiv u_i - \overline{u}_i$$

And repeated indices imply summing over.

Let us for the sake of generality assume that **λ** is of dimension **γ** and that the first **β** values of these are measured from data. Tnen we need to determine **γ-β+1** remaining parameters by minimizing χ^2. Minimizing with respect to the unknown parameters, we get

$\frac{\partial\chi^2}{\partial\lambda_i} = 2H_{ij}\lambda_j = 0$; where **i** runs from **β+1** to **γ** and j runs from 1 to γ. This leads to

$H_{ik}\lambda_k = -H_{il}\lambda_l$ where both i and **k** run from **β+1 to γ** and the subscript **l** runs from **1 to β**. The unknown parameters **λ_k** can be obtained by inverting the square submatrix **H_{ik}**.

Application to our problem

In our case we have one unknown, **q** and four measurements **x,x',y,y'**. We need to study the correlations between the momentum **q** and the four measurements. It may turn out that the correlation with **x** variables is weak. In which case, we may work in **y** space entirely. It should be noted that the above technique is quite general and useful for solving for unknown parameters using our knowledge of the model in a linearized fashion.